

Problem 3.

Soln.

let $X \sim U(0,1)$, pdf of X

$$f_X(x) = 1 \quad ; \quad 0 < x < 1$$

$$f_{Y|X} \sim r(0, x)$$

$$f_{Y|X}(y|x) = \frac{1}{x} \quad ; \quad 0 < y < x \quad ; \quad 0 < x < 1$$

$$\text{if } f_Y(y) = -\ln(y) \quad 0 < y < 1$$

$$* f(y|x) = \frac{f(x,y)}{f(x)}$$

$$f(x,y) = \frac{1}{x} \cdot (1)$$

$$f(x,y) = \frac{1}{x} \quad ; \quad 0 < y < x$$

$$(1) f_{x|y}(x,y) = \frac{f(x,y)}{f_Y(y)} = \frac{\frac{1}{x}}{-\ln(y)}$$

$$f_{x|y}(x,y) = \frac{-1}{x \ln(y)} \quad ; \quad y < x < 1$$

$$(2) h(y) = E(x|y)$$

$$= \int_y^1 x \frac{-1}{x \ln(y)} dx$$

$$= \frac{-1}{\ln(y)} [x]_y^1$$

$$= \frac{-1[1-y]}{\ln(y)}$$

$$\Rightarrow h(y) = \frac{-(1-y)}{\ln(y)} \Rightarrow \underline{\underline{\frac{y-1}{\ln(y)}}}$$

Problem 5.

1. $E(Y_1)$

$$Y_1 = X_1 + N$$

$$E(Y_1) = E(X_1) + E(N)$$

$$E(N) = 2$$

$$E(X) = 1 \times q + (1-q)$$

$$E(X) = q$$

$$E(X) = \int_0^1 q \, dq$$

$$= \frac{q^2}{2} = \frac{1}{2}$$

$$\therefore E(Y_1) = \frac{1}{2} + 2 = \underline{\underline{5/2}}$$